Real World Trigonometry:

Surveying

December 2009

Lesson Overview

In the course of this 2-3 week lesson the student will learn to use basic surveying techniques in order to realize the real world practical applications of trigonometric principles. They will first learn terminology and practices on graph paper using a compass and straight edge; then they will practice using the magnetic compass on the schoolyard. The culmination of the project will be a where grouped students, using surveying tape measure and magnetic compass, will negotiate a complete traverse on the schoolyard verifying their results mathematically.

Note to teacher. This lesson--especially the worksheets--will include details based on the location of the writer of the lesson. Each teacher will have to personalize many of the details of bearings and distances used in this lesson to their individual locations.

Materials Included in this Lesson

- Lesson overview for each day
- Worksheets
    1. Plot angles, find distances from the origin using trig functions, and determine the x,y coordinates for a given distance and bearing.
    2. Plot angles, find distances and determine the x,y coordinates for a given distance and bearing from places other than the origin.
    3. Plotting angles and distances on graph paper including a full traverse.
    4. Compass practice I: Find bearings and distances to school objects on the school yard and then graph.
    5. Compass practice 2: Making a full traverse on schoolyard.

Other Materials needed for this Lesson

- 1 protractor per student
- 1 ruler per student
- 1 magnetic sighting compass per group
- 1 surveying measuring tape per group (surveying measuring tape has feet divided into tenths in addition to 12ths)
- 1 graphing calculator per group--or a calculator that will allow the students to do inverse trig functions.
- Engineering Graph Paper

Skills the Student will Learn

- Vocabulary:
  - Azimuth
  - Traverse
  - Control
  - Back site
  - Chain
  - Bearing
  - Set a point
  - Occupy a site
  - Transit

- To read a quadrant graph in the clockwise direction as bearings on a compass.
- To plot angles on graph paper and using trig functions to find exact (x,y) coordinates for points.
- To use a compasses to find given bearings.
- To take precise measurements and double check to verify accuracy.
- To use a surveying measuring tape to measure in decimal increments.
- Map out a minimum of 3 objects on graph paper giving bearings, distances and coordinates precise to tenths.
- Graph a full traverse and verify the measurements using the law of sines and/or the law of cosines.

Student Deliverables

- 3 worksheets filled out
- Graph of objects in the schoolyard with bearings,distances and xy coordinates labeled.
- Graph of the traverse on the schoolyard with another sheet with all of the math calculations.

Length of Lesson: 8 Days

Day One: From a transit given a distance and bearing find coordinates

Synopsis: Introduce surveying (20 min). Talk about how surveying is used in the real world, in both vertical and horizontal planes. Without repeated surveying at each large jobsite (or a one time shot at simple family home site) based on fixed points--either set arbitrarily or by GPS to exact latitude and longitude--one small discrepancy can be multiplied at each step of the construction process until finally, after many steps in the process, a building can be structurally unsound. But how do we set absolute points and positions? Hand out the vocabulary worksheet. Drawing on a graph on the whiteboard, define each of the terms. Emphasize how to read the coordinate graph with the four quadrants and how to write bearings.

On a coordinate graph show the student how to turn an angle with a bearing in the first quadrant, and a distance. From the given information, show the students how by using trig functions, one can then determine the x and y coordinates. Then turn another angle in another quadrant and determine the xy coordinates. Pass out the worksheet and ask students to do the first problem by themselves. When most of the class has finished the problem, do it on the board. Homework is the rest of the worksheet.

Day Two: Plotting a traverse on graph paper

The goal of this lesson is to plot a traverse from origin back to origin. In this lesson we will limit ourselves to triangular traverses.

First, the teacher reviews the Law of Cosines. Teacher emphasizes that the law of cosines is applicable when you are working with triangles that are not right. He draws out a traverse on the board and emphasizes that the double check of a 3 point triangular traverse is that the angles sum to 180º. The teacher demonstrates a triangle on the board. The traverses for the first and second points and distance are given.

Using the law of cosines, the teacher demonstrates that the third side and all of the angles can be found. For example, if the teacher gives the first traverse as N300E, 100', and the second point is S200W at 90'. From this we can figure out that the angle between two given sides must be 50º. So we can apply the Law of cosines to find the side opposite the known angle: $c^2 = 100^2 + 90^2 - 2(100)(90)2\cos 50$ so that we find that c = 80.8. From here, using, again, th law of cosines and the inverse trig functions, the student can find one of the other angles in the triangle and by subtraction, the third angle of the triangle. Once the students have found the three sides of the triangle, The teacher then refers the students to the first problem on the worksheet in which the first two traverses are given and the students must find the length of the third side and measures of the angles. After the students have worked out their answers, the teacher demonstrates how to do the problem on the board. The students then are given time to complete the rest of the worksheet. Before they begin, the teacher again emphasizes that the surveyor after doing their work double checks by making sure that all of the values work for each of the angles--that is, they do the law of cosines three times. The worksheet gives problems with the three angles and lengths and asks for the student to verify whether the surveyor has done his job well or badly--i.e., are the measurements accurate?

Day Three: Compass Practice

Compass practice. The students are given the sighting compasses. They are given 5 minutes just to play with them.

Teacher passes out the surveying measuring tapes. The students are asked to describe the difference between this measuring tape and the ruler that they are accustomed to use. The students tell the teacher that the feet are broken into tenths rather than 12ths. There is a short discussion on why this might be used rather than the usual 12ths.

After this short discussion the teacher draws a control line on the board. He/she gives the students the worksheet with step-by-step instructions on how to find the bearings of various objects on the schoolyard (see the charting schoolyard worksheet). Students are to use the compasses and the tapes to find objects on schoolyard given heading and distance from an origin point.

Activity Day Four

This day, the students plot a traverse of two objects on the yard and back to the control point continuing on the worksheet that they got the day before. After they are finished, they are to draw a graph and double check their measurements by using the law of cosines. The final product is a map of the three objects on the schoolyard and the calculations to confirm that the measurements were accurate. At the end of this day's activity, the groups should turn in a completed map of their traverse. In addition, there should be a diagram of the traverse with the triangle labeled with lengths and angles and the calculations of the law of cosines presented in a clear form.

Activity Day Five

Out in park, students are given bearings and distances and they have to plot the objects they find onto graph paper. The measurements worksheet should have a section for notes. Teacher must emphasize that it is VERY important to note ANY discrepancies or anomalies as they work so that anyone could follow what they do exactly. Score is given for accuracy. The groups have to elect a party chief and chainman and engineer. Roles: Party chief runs the compass. Chainman takes care of moving and measuring. Engineer is responsible for the drawing. Scores are for measurements and for drawing. Thus the measuring folks aren't graded for bad drawing and engineer is not penalized deeply for bad measurements.

Activity Day Six

Each group is given a set of objects and they have to find bearings and distances to make a traverse back to origin. This is a competition. The majority of the grade is based on accuracy. The rest of the grade is given for speed. Each of them is given 5 objects to traverse. They must map the objects on graph paper. Each must give xy coordinates for each object and angles and distances.

Activity Day Seven

Finish Map and submit.

Activity Day Eight

Class writes reflection of activity and submits final work folder with each of the worksheets and maps and reflection.

Enrichment Suggestions

One possible extension to the lesson would be to have the students make a control line that is NOT true north and then compute bearings and distances based on a random control line.

Another enrichment activity would be to look at the equation for determining the sum of the interior angles of any n-sided polygons in order to double check the accuracy of a traverse.

State Standards Met

Lesson meets the following State Standards: Trigonometry: 2.0, 8.0, 12.0, 13.0, 19.0.

Rubric for the Surveying Project

Student Deliverables	1 Exceeds Expectations	2 Meets Expectations	3 Approaches Expectations	4 Fails to meet Expectations
Worksheet 2	Student has met the expectations and, additionally, has graphed to scale. The graphs are clearly labeled. The student's math work to find the x,y coordinates is clean and clear and accurate.	Student has accurately graphed the bearings and lengths. Student has accurately found the x,y coordinates for each point. The work is accurate though not necessarily "clean". The graph is close to scale.	Student has attempted to represent the point on the graph with some errors. The drawings are not necessarily to scale. Student has some errors in his/her computations. Work is messy and confusing.	Student does not locate points accurately on the graph Student does not derive the x,y coordinates. The work is not to scale.
Worksheet 3	Student has met the expectations and, additionally, has graphed to scale. The graphs are clearly labeled. The student's math work to find the x,y coordinates is clean and clear and accurate.	Student has accurately graphed the bearings and lengths. Student has accurately found the x,y coordinates for each point. The work is accurate though not necessarily "clean". The graph is close to scale.	Student has attempted to represent the point on the graph with some errors. The drawings are not necessarily to scale. Student has some errors in his/her computations. Work is messy and confusing.	Student does not locate and scale points accurately on the graph Student does not derive the x,y coordinates. The project is not completed.
Charting schoolyard: Day1	Student has met the expectations and, additionally, has graphed to scale. The graphs are clearly labeled. The student's math work to find	Student has accurately graphed the bearings and lengths. Student has accurately found the x,y coordinates for each point. The work is accurate	Student has attempted to represent the point on the graph with some errors. The drawings are not necessarily to scale. Student has some errors in	Student does not locate and scale points accurately on the graph Student does not derive the x,y coordinates. Student does not work efficiently and

	the x,y coordinates is clean, clear and accurate and done on a second sheet. Everything is clearly labeled. The work is presentation worthy.	though not necessarily "clean". The graph is close to scale. Student worked efficiently and professionally when on the schoolyard. There is a final draft. Points are clearly labeled including scale. Math is accurate.	his/her computations. Student is mostly efficient and professional when taking measurements. Work is messy and confusing. The work may be in rough draft form.	professionally when taking the measurements. The project is not completed.
Surveying a Traverse on the Schoolyard	Student has met the expectations and, additionally, has graphed to scale. The graphs are clearly labeled. The student's math work to find the x,y coordinates is clean, clear and accurate and done on a second sheet. Everything is clearly labeled. The work is presentation worthy. All of the live measurements are verified mathematically and are accurate.	Student has accurately graphed the bearings and lengths. Student has accurately found the x,y coordinates for each point. The work is accurate though not necessarily "clean". The graph is close to scale. Student worked efficiently and professionally when on the schoolyard. There is a final draft. Points are clearly labeled including scale. Math is predominantly accurate.	Student has attempted to represent the point on the graph with some errors. The drawings are not necessarily to scale. Student has some errors in his/her computations. Student is mostly efficient and professional when taking measurements. Work is messy and confusing. The work may be in rough draft form. The measurements are marginally verified mathematically, or are inaccurate.	Student does not locate and scale points accurately on the graph Student does not derive the x,y coordinates. Student does not work efficiently and professionally when taking the measurements. The project is not completed. The measurements are not verified mathematically.

Vocabulary for Surveying

Azimuth: The north south axis. Note: the azimuth is NOT absolutely linked to magnetic north. It CAN be set up that way. But the surveyor often determines the azimuth based on the site geography.

Traverse: To close the circuit of bearings/distances measurements.

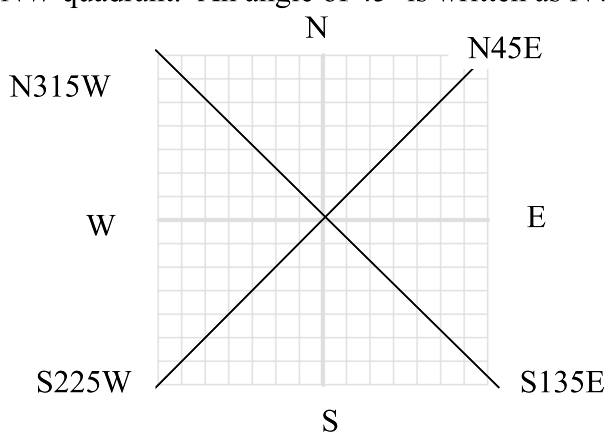
Control: The original line against which all measurements are checked.

Back site: To confirm the initial measurement. Once the point has been set, the transit is moved to the set point and the chainman occupies the control and confirms that the angle turned is the same.

Chain: The distance measurement tool from the set point to the occupied site.

Chainman: The person who uses the surveyors tape to measure out from the transit in order to set a point.

Bearing: Bearings are based on a compass. The quadrants are the opposite of standard geometry. First quadrant is the same. But then we count the quadrants in a clockwise direction. The positive Yaxis is north. Positive X axis is east. The way to write the bearing is based on the quadrant. 0-90 degrees is in the NW quadrant. An angle of 45º is written as N45E.



Set a point: The point measured from the transit at a given distance and bearing.

Occupy a site: Place the transit on the control point.

Transit: The occupied site, that is the point from which the surveyor calculates the bearing.

Surveying Unit Worksheet 1: Turning angles from the Origin

For each of the following problems:
1. Graph the bearing and distance.
2. Determine the (x,y) coordinates of the set point using sine & cosine functions.

1. N45E; 1 cm

2. S225W; 1 cm

3. S150E; 1.5 cm

4. N300W; 1.5 cm

5. S128E; 1.8 cm

6. S259W; 0.7 cm

7. N24.7E; 0.35 cm

8. N 305.8W; 1.37 cm

Surveying Unit Worksheet 1: Turning angles from the Origin

For each of the following problems:
1. Graph the bearing and distance from the given point.
2. Determine the (x,y) coordinates of the set point using sine & cosine functions.

1. N45E; 1 cm, (1,1)

2. S225W; 1 cm, (-2,-2)

3. S150E; 1.5 cm (-4, 3)

4. N300W; 1.5 cm (3,-5)

5. S147.5E; 1.8 cm (-5, -2)

6. S229W; 0.7 cm (4,1)

7. N24.7E; 0.35 cm (-10, -2)

8. N 305.8W; 1.37 cm (15, -5)

Using your Compass
Determining Bearings of Objects on School yard
Based on a control line--Day One.

Day One

Step 1: Choose a point at some distance (minimum 50 feet. Sight the object at the hair on your compass and move until the N and the 0 line up. Make sure that you level the compass by making sure that the bubble is centered over the center of the compass glass. Mark the point where you're standing. This will be your control point.

Step 2: Go to the northern object and using a place-marker at the original control point (this can be another student in your group). Double-check that your control point is exactly due south. This step is important because you will be relating all of your measurements to this azimuth, or control line, and so you have to make sure that your control line is exact. Otherwise all of your other measurements will be off.

Step 3: Return to your control point and choose an object on the schoolyard.

Step 4: Level the compass--if necessary; make sure that the bubble is centered as much as possible at the top of the compass.

Step 5: When you've sighted the object and the compass is level, read the bearing on the compass. On a magnetic compass, the arrow will always point to the north. Point the compass to the new object. Compute the difference between the angle measurment on the compass, and true north. Send out the chainman to measure the exact distance between the control point and the object.

Step 6: Re-sight the control object due north to recalibrate the compass. Then take the bearing of your chosen object a second time to double check the reading. THIS STEP IS VERY IMPORTANT.

Step 7: Now, with your chainman standing at the control point, let the transit (i.e. the compass) be at the object and backsite your compass reading. With the bubble centered on the compass, your backsite should be 180º different from your original reading. If it is not, redo your measurements until you have a correct bearing.

Step 8: Repeat steps 1 through 5 and find the bearings and distances of 5 objects on the schoolyard. Three of these objects must be: One of the basketball poles, and two building corners. You may choose two other objects. But, your objects must be no more than 100' from your control point.

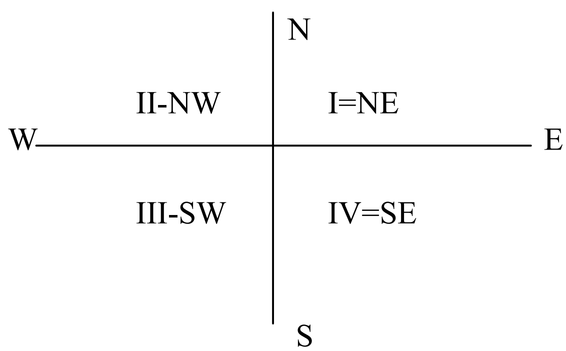
Step 9: Using graph paper, make a SCALE drawing with the bearings and distances of the objects on the yard, using your control point as the origin of the xy graph. You must specify the exact (x,y) coordinates for each object. EACH GROUP MUST PRODUCE A PERFECT GRAPH WORTHY OF PUBLIC EXHIBITION with all of the bearings and distances and (x,y) coordinate measures clearly and neatly delineated.

Computing a Traverse

One exercise in Surveying is to compute a complete traverse. A traverse takes a surveyor from a starting point through a series of angles back to the point of origina.

First, here’s some surveying vocabulary:

- Azimuth: The original north south line.
- Traverse: A completely surveyed circuit from the control point through several angles and back to the original control point
- Control: The original point from where one begins to survey: the origin. Also, the control can be the original true north line passing through the control point.
- Back site:
- Chain: The measurement tool from the transit to the surveyed point.
- Bearing: The direction of the angle turned. The bearing is measured *clockwise* around the coordinate axis--***NOT*** counterclockwise as we usually measure angles. The names of the four quadrants are as follows:



- Set a point: Determine an absolute point by double-checking against the control.
- Occupy a site: Set up the transit at a given point.
- Transit: The “compass” from which will turn the angle and set the next point.

Directions:

From the origin mark an angle N60E at 22.5 meters long. This will be your new occupied site. From this occupi site, mark off an angle N285W 43.3 meters long. Now given these two measurements, find:

The angle from the second occupied point back to the control.
The length of that last chain.

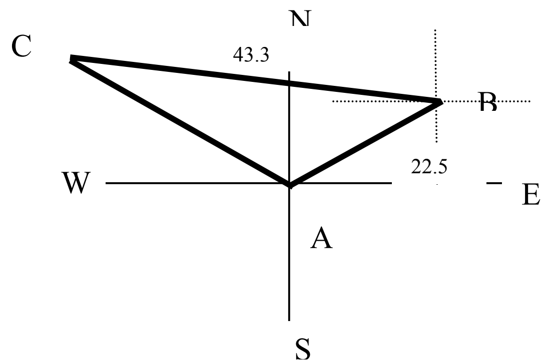
The final measurements of the triangle you made on your traverse.

You will have to use the law of cosines and the law of sines to solve this problem.

$$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$$

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

Here is the solution to the problem:



Angle NAB = 60º
CB = 43.3
AB = 22.5

ABC = 45º
(30º + 15º)

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$b^2 = 43.3^2 + 22.5^2 - 2(22.5)(43.3)(\cos 45)$$

$$b^2 = 1874.89 + 506.25 - 1377.8$$

$$b^2 = 1003.34; \rightarrow b = 31.67$$

Therefore, CA = 31.67

Now, using the law of Sines:

$$\frac{\sin C}{c} = \frac{\sin B}{b} = \frac{Sin A}{a} \rightarrow \frac{\sin 45}{31.67} = \frac{\sin C}{22.5} \rightarrow \frac{22.5\sin 45}{31.68} = \sin C \rightarrow \sin C = 0.5022 \rightarrow \sin^{-1} 0.5022 = 30.15°$$

Now to Verify:

$$180° - 45° - 30.15° = 104.85$$

$$a^2 = b^2 + c^2 - 2bc\cos A \rightarrow 43.3^2 = 22.5^2 + 31.67^2 - 2(22.5)(31.67)\cos 104.85$$

$$\frac{43.3^2 - 22.5^2 - 31.67^2}{2(22.5)(31.67)} = \cos 104.85 \rightarrow\rightarrow 0.2566 = 0.2563$$

Therefore, QED.

This same process will yield solutions to the other problems.

Three more Traverse problems

Given the following measurements find:
The angle from the second occupied point back to the control.
The length of that last chain.
The angles of the triangle created by the traverse.
The x,y coordinates of each of the points.

1. N58E, 112m then S212W 205 m

2. S262W 25m then S112E 20m

3. N321W 87.5 m then S181W 92m

Using your Compass
Determining Bearings of Objects on School yard
Based on a control line--Day Two

<u>Day Two</u>

Step 1: Today return to your control point. Reconfirm your control line by repeating steps one and two from day one.

Step 2: Find the bearing from your control point to one of the objects. Have the chainman measure the distance from the control point to the object. When you reach that object, double check the bearing from that point back to the control point. If you did your measurements correctly, the second bearing should be 180º plus/minus of the bearing from control to the object.

Step 3: Once you've set this point, it becomes your control point. This means that you need to find another Due North point at some distance away. Double-check this measurement by moving to the north point and confirming that the new control point is due south from the new north point.

Step 4: Once you've established this new control line, take the bearing to the second object. Have the chainman measure the distance between the first and second objects.

Step 5: Repeat step 3 from your new set point in order to establish a new control line.

Step 6: From this new control line measure the bearing back to your starting point. Have your chainman measure the distance from this third point back to your original control point.

Step 7. From your original control point, take the bearing to the second point of your traverse. This bearing should be 180º different from the bearing in step 6.

Final process. Map out your traverse TO SCALE on graph paper. Include the math using the Law of Cosines to confirm that your physical measurements were accurate. You must specify the exact (x,y) coordinates for each object. EACH GROUP MUST PRODUCE A PERFECT GRAPH WORTHY OF PUBLIC EXHIBITION with all of the bearings and distances and (x,y) coordinate measures clearly and neatly delineated.

Vocabulary for Surveying

Azimuth: The north south axis. Note: the azimuth is NOT absolutely linked to magnetic north. It CAN be set up that way. But the surveyor often determines the azimuth based on the site geography.

Traverse: To close the circuit of bearings/distances measurements.

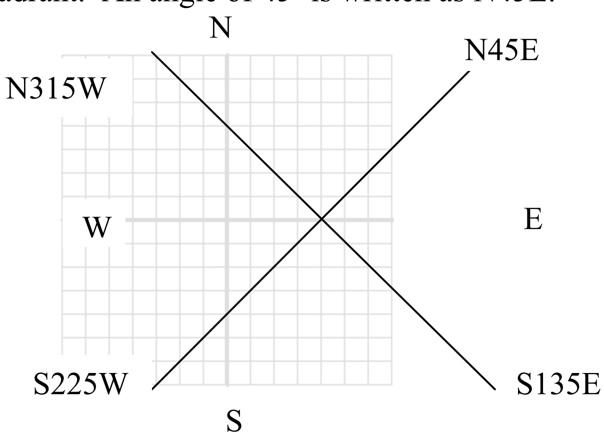
Control: The original line against which all measurements are checked.

Back site: To confirm the initial measurement. Once the point has been set, the transit is moved to the set point and the chainman occupies the control and confirms that the angle turned is the same.

Chain: The distance measurement tool from the set point to the occupied site.

Chainman: The person who uses the surveyors tape to measure out from the transit in order to set a point.

Bearing: Bearings are based on a compass. The quadrants are the opposite of standard geometry. First quadrant is the same. But then we count the quadrants in a clockwise direction. The positive Yaxis is north. Positive X axis is east. The way to write the bearing is based on the quadrant. 0-90 degrees is in the NW quadrant. An angle of 45º is written as N45E.



Set a point: The point measured from the transit at a given distance and bearing.

Occupy a site: Place the transit on the control point.

Transit: The occupied site, that is the point from which the surveyor calculates the bearing.

Surveying Unit Worksheet 1: Turning angles from the Origin

For each of the following problems:
1. Graph the bearing and distance.
2. Determine the (x,y) coordinates of the set point using sine & cosine functions.

1. N45E; 1 cm

2. S225W; 1 cm

3. S150E; 1.5 cm

4. N300W; 1.5 cm

5. S128E; 1.8 cm

6. S259W; 0.7 cm

7. N24.7E; 0.35 cm

8. N 305.8W; 1.37 cm

Using your Compass
Determining Bearings of Objects on School yard
Based on a control line--Day One.

Day One

Step 1: Choose a point at some distance (minimum 50 feet. Sight the object at the hair on your compass and move until the N and the 0 line up. Make sure that you level the compass by making sure that the bubble is centered over the center of the compass glass. Mark the point where you're standing. This will be your control point.

Step 2: Go to the northern object and using a place-marker at the original control point (this can be another student in your group). Double-check that your control point is exactly due south.

Step 2: Return to your control point and choose an object on the schoolyard.

Step 3: Level the compass--make sure that the bubble is centered as much as possible at the top of the compass.

Step 4: When you've sighted the object and the compass is level, read the bearing on the compass. Send out the chainman to measure the exact distance between the control point and the object.

Step 5: Re-sight the control object due north to recalibrate the compass. Then take the bearing of your chosen object a second time to double check the reading. THIS STEP IS VERY IMPORTANT.

Step 6: Repeat steps 1 through 5 and find the bearings and distances of 5 objects on the schoolyard.

Step 5: Using graph paper, make a SCALE drawing with the bearings and distances of the objects on the yard.

Using your Compass
Determining Bearings of Objects on School yard
Based on a control line--Day Two

Day Two

Step 1: Today return to your control point. Reconfirm your control line by repeating steps one and two from day one.

Step 2: Find the bearing from your control point to one of the objects. Have the chainman measure the distance from the control point to the object. When you reach that object, double check the bearing from that point back to the control point. If you did your measurements correctly, the second bearing should be 180º plus/minus of the bearing from control to the object.

Step 3: Once you've set this point, it becomes your control point. This means that you need to find another Due North point at some distance away. Double-check this measurement by moving to the north point and confirming that the new control point is due south from the new north point.

Step 4: Once you've established this new control line, take the bearing to the second object. Have the chainman measure the distance between the first and second objects.

Step 5: Repeat step 3 from your new set point in order to establish a new control line.

Step 6: From this new control line measure the bearing back to your starting point. Have your chainman measure the distance from this third point back to your original control point.

Step 7. From your original control point, take the bearing to the second point of your traverse. This bearing should be 180º different from the bearing in step 6.

Final process. Map out your traverse TO SCALE on graph paper. Include the math using the Law of Cosines to confirm that your physical measurements were accurate.